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Period Doubling Occurences in Wind Instruments Musical Performance

Vincent Gibiat
Laboratoire Ondes et Acoustique ESPCI/Université Paris 7, UMR CNRS 7587, 10 rue Vauquelin, 75231 Paris Cedex 05, France

Michèle Castellengo
Laboratoire d'Acoustique Musicale, UMR 7604 CNRS-Ministère de la Culture - Université Paris 6, Case 161, 4 place Jussieu, 75252 Paris cedex 05, France

Summary
As a special class of Non Linear Dynamical Systems, self sustained musical instruments can exhibit peculiar behaviours related to bifurcation and chaos theories. While many quasi periodic occurrences in woodwinds and bowed strings have been reported, only a few period doubling scenarios have been observed in musical instruments, and most of the time they have been obtained through numerical simulations or specially designed hybrid experimental set-ups far from musical situations. We present here a set of experimental results, all related to this period doubling scenario and always in direct connection with musical performance on wind instruments and voice. We show that at least three period-doublings can be obtained on a trombone, that the crumhorn and the bassoon can oscillate in the same way and that traditional singers, and in some cases classical singers, exhibit phenomena clearly linked with this scenario. Finally, from the whole set of experimental results available for this particular set of multiphonic sounds we give some hints for a general schema governing their musical production.

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1. Introduction

The description of wind instruments as a special case of Non Linear Dynamical Systems (NLDS's) has become nowadays as classical as the linear description was some decades before. Numerical simulations and theoretical studies as well as experimental works on woodwinds and voice have been done which have proved the validity of this description [1, 2, 3, 4, 5, 6].

The basic tools needed for understanding NLDS's have been known for at least a century and include linear expansion and stability analyses at threshold [7]. These concepts allow the study of self sustained musical instruments in terms of bifurcations and transitions to chaos [8, 9, 10].

Among the three classical scenarios known to be precursors of chaos, only quasi periodicity [11] has been widely met in musical signals produced by musicians [3, 12, 4, 5]. This scenario predicts that after a first bifurcation the behaviour of the instrument is described by a limit cycle in its phase or state space, which corresponds to the normal periodic sound of the instrument, and that a second bifurcation may occur which may lead to a more complicated trajectory. This trajectory may be a torus or a periodic trajectory with a period doubling. In the first case, a third bifurcation may give rise to a chaotic state described by a strange attractor in the phase space. We do not discuss here in detail the different possibilities, (-3-torus involving a third frequency or Curry and Yorke model with only two frequencies-); the reader can find in literature and references the information he needs. The torus trajectory may be connected to a periodic orbit in phase space in case of a phase locking when the two (or three or even more) frequencies are in a rational ratio [8]. More generally the behaviour of the system corresponds to a quasi-periodic state. This means that there is no simple rational ratio between the base frequencies that have give rise to the torus. A very important point is that this scenario can only lead to chaotic behaviour under very restrictive conditions about the implicated base frequencies. The most important one is the non-harmonic relation that is necessary between the base frequencies. This point is somehow in contradiction with the skill that woodwind makers prove in obtaining quasi-harmonic relations between the resonances of their instruments. Quasiperiodicity is a very common state in mechanical, acoustical and musical systems, but in the case of musical instruments it has never been proved that it may lead to chaos, even if some hints of chaotic features have been noticed. Triperiodic and quadriperiodic states have been obtained, as is theoretically possible, when there is a simple rational relation between the three base frequencies [13, 5, 14]. For this scenario, the determination of Lyapunov exponents [14, 9] remains the only valid way to prove the presence of chaos. Unfortunately, as far as we know, it has never led to this conclusion for such systems.

On the other hand the most widely studied scenario, known as period doubling scenario [15, 16, 17], which always yields chaos, has only been demonstrated for numerical and experimental set-ups mimicking woodwinds, [18, 2, 3, 12] which remain far from true musical situations. A few examples of period doubling in woodwinds have been presented on an isolated edge tone [19] or on trumpet tones [20] but, as far as we know, there is no example in scientific literature of a more complete cascade in a musical situation. The first report in musical acoustics literature of such a phenomenon, produced

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by a musician and explicitly related to the period doubling scenario, is a recent work on the bassoon [21].

The aim of this paper is to show that period doublings may be met in a great number of musical situations, all concerning classical or renaissance musical instruments played by experienced musicians or singers. It is not our purpose to compute the Lyapunov exponents or the correlation dimension on chaotic signals or to give a complete theoretical or numerical model. The following experimental results are presented here: a cascade of period doublings obtained by a trombonist, playing at a high level (what he calls "octave multiphonics") the same phenomenon in various double reed instruments and finally in the voice. All signals have been recorded in reproducible laboratory conditions with the exception of the two ethnic musical examples.

A special case of musical wind instrument is that of the human voice. For this particular mechanical and musical system, the period doubling occurrence has been known for years [22, 23, 24, 25] and has been numerically simulated and theoretically studied by numerous researchers [26, 27] through two-mass models, for example. Nevertheless these occurrences did not concern musical performance but voice disorders or vocalisations by newborn children. We give here two examples of period doubling in traditional singing that can be compared with the experimental data recorded with a singer who has developed a great skill in this kind of low-pitch voice production.

The signals have been analysed both with the classical tools of signal processing and those of Non Linear Dynamical Systems (mainly delayed Phase Space Representations -PSR- and Poincaré Sections), and by direct sonagraphic observation. The first type of analysis has proved to be powerful in situations [3] where the duration is long enough and where the harmonic content is not too high. In all other cases the spectrographic analysis has been used. It gives a direct access to a spectral bifurcation diagram. We do not give here any detailed information about the properties of NLDS analysis that are beyond the scope of this paper; only the main useful basic ideas are presented. The reader is invited to refer to specialised publications [8, 10] for more information.

Enlightened by these experimental analyses we then give some hints for a qualitative explanation of such behaviours, and we propose some possible solutions to obtain such period doubling cascades in other instruments such as clarinet or saxophone.

2. Period doublings in a trombone

When playing multiphonic sounds [28, 29], experienced musicians mix a great deal of know-how and a lot of knowledge. Since nothing is nowadays absolutely established and taught in music schools concerning this particular class of musical sounds (even if catalogues of multiphonics exist for some instruments), each player has a particular set of multiphonics and uses his personal technical solutions to produce them. On woodwinds, it is often stated that multiphonics are always defined by special fingerings which give non harmonic

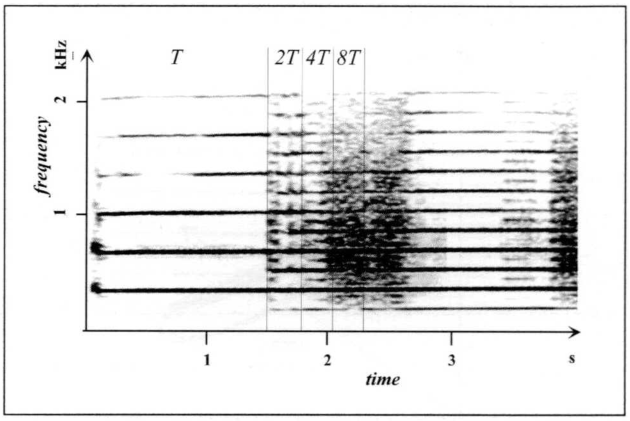

Figure 1. Sonagraphic representation of a trombone octave multiphonic. This representation should be understood as a spectral bifurcation diagram. The bifurcations points are represented by thin vertical lines.

resonances [30]. Musicians have known for a long time and it has been proved [12, 5] that normal fingerings are able to produce multiphonic sounds as well as special fingerings even if the acoustical resonances are harmonically related. Moreover the same fingering does not always give the same perceived multiphonic sound or the same spectral content. The main characteristic of multiphonics is the wide variety that can be obtained through changes in the main parameter that is under the control of the musician: the embouchure. It is particularly true for instruments where no fingerings are used, as is the case for the trombone.

One of the authors was extremely lucky to work with a trombonist of the "Ensemble Intercontemporain" [31], and to record under his control a wide collection of sounds. This exceptional musician is able to produce an important number of different multiphonics: most of them are quasiperiodic, or quasi periodic with phase locking [29], but some others, which he calls "octave multiphonics" do not correspond to this description.

To obtain these peculiar sounds the musician begins to play a periodic sound on an upper partial of the instrument (in the presented example it is the sixth partial corresponding to the note F4, but it can also be obtained on another partial). After this normal sound, he changes his embouchure and obtains something like the third partial of the instrument, approximately one octave lower; then he bifurcates once more and produces a rough sound that should to be considered again as a sound one octave lower. After another embouchure modification he produces what appears to be, after spectral analysis, a sound with a missing fundamental lower than those normally possible on a trombone with this position of the slide. This (missing) fundamental frequency does not correspond to a resonance of the instrument. The sonagraphic analysis presented on Figure 1 shows clearly this cascade of bifurcations. A first division by two of the fundamental frequency (period T) is immediately followed by a division by two (period $2T$) and then by less obvious divisions, which may correspond to other period-doublings. It suggests that the sound production of these «octave mul-

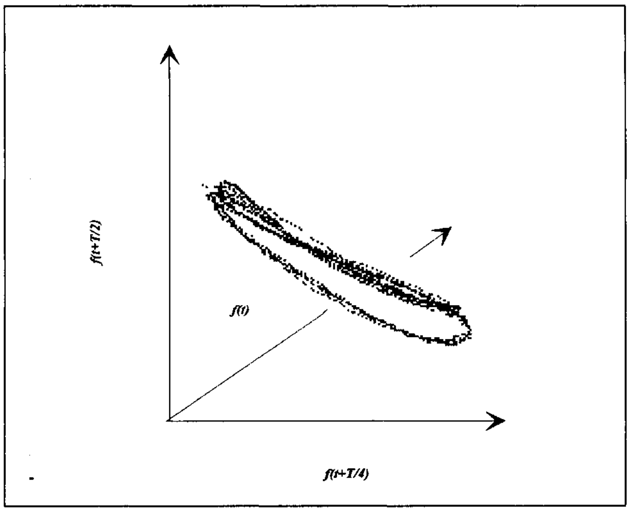

Figure 2. Reconstructed delayed Phase Space Trajectory of a periodic signal of trombone. The delay chosen is one quarter of the period of the fundamental frequency. This general behaviour very near to a closed curve is obtained for the sustained part of woodwinds if their harmonic content is small enough.

tiphonics» can be interpreted as a period-doubling cascade from the sixth partial of the instrument. As the final sound does not necessarily correspond to any resonance of the instrument the level of the lowest components of the sound is very weak. These components often appear only as traces on the sonagraphic representation. This first analysis, which corresponds to a spectral bifurcation diagram, can be improved and confirmed with another kind of representation in phase space.

In a phase space representation (PSR) a periodic signal gives a closed curve, that is, a strictly periodic signal can be represented as the limit cycle of a simple oscillator on a 2-D surface. Under normal playing conditions, stable sounds of musical instruments give such curves (Figure 2). This normal and periodic sound has been obtained after a first bifurcation and is of frequency F. The period-doubling scenario replaces this periodic signal by another one showing a fundamental frequency divided by two. This occurs at the bifurcation point. Since the frequency of a nonlinear system basically changes when increasing the control parameter, the new period is not exactly twice the original one but twice that of the fundamental frequency just before the bifurcation which may slightly differ from F.

How can we distinguish a signal produced by a system which has bifurcated and whose period has doubled from a signal produced by a system in which the second harmonic is more intense than the first one? Formally it does not seem possible. The trajectory of a "first period doubling" and that of a periodic limit cycle in which the second harmonic is very strong have similar shapes. Nevertheless on most physical systems it remains true that a limit cycle, for infinitely small amplitudes, corresponds either to a quasi-sinusoidal, or to one presenting a strong first harmonic. This is the case of the square oscillations obtained by Maganza *et al.* [2] or Grand [32]. Moreover, at the bifurcation point, the spectrum of a period doubling always presents very weak components corresponding to the odd multiples of the lowest component $F/2$. So an oscillation with frequency $F/2$ following an oscillation at frequency F with a very strong second harmonic $2F$ or close to it, should be interpreted as a first period doubling. It seems that it corresponds to the situation represented in Figure 3a. This assumption is well confirmed by the other bifurcations (Figures 3b and 3c). The third period doubling is not as clear as the previous one but it is visible as a folding of the trajectory (Figure 3c). The last part of the signal does not give any more information. It is very difficult to identify other foldings on the PSR and it is absolutely impossible to say whether the musician reaches a chaotic situation without computing the Lyapunov exponents. Since the signals are too short and basically unstable for the musician, such a determination is unfortunately beyond our possibilities. Anyway one must keep in mind that the period doubling scenario is a clear indication of a route to chaos whenever a period tripling is detected.

How does the musician produce these sounds? First it is important to note that this kind of sound is fundamentally different from what is obtained when the musician is singing in the instrument. There is no production through the vocal tract of any sound one octave lower. There is only one excitatory system. A woodwind player can control one easily measurable parameter, the blowing pressure, and many others that are very difficult to evaluate. All these parameters together define what we call the embouchure. It includes the position of the lips, their tension, their opening and many other "physical" parameters that are difficult or impossible to measure such as the lips, and all their mechanical properties (mass, stiffness, damping...). So we have to trust in the musician's sensations. The only thing he is able to say is that he obtains these sounds by relaxing the embouchure.

3. Period doubling on reed instruments

Some reported results on double-reed instruments may also be interpreted as possibly chaotic features or period doublings, even if the authors who presented these results did not give such interpretations. For instance, some "doubled signals" can be found in Barjau [33]. It is also known from organ-maker's knowledge that organ pipe reeds with a bad curvature sound at a lower frequency than expected but with a poor musical quality [34]. Figure 4 presents the spectrographic representation of the sound produced by a bad adjustment of the reed of a Cliquot organ pipe at the Cathedral of Poitiers (E4 'hautbois du récit'). The tone is unstable and bifurcates quickly to a perfectly audible period doubling. These observations suggest that period doubling cascades may be possible on reed instruments.

We have chosen to experiment on a very simple instrument, the crumhorn, whose main advantage is the absence of contact between the musician and the double reed. Another interest of this unusual Renaissance instrument is that the reed is placed in a small cavity. This small box has resonance frequencies higher than those of the bore. Using a

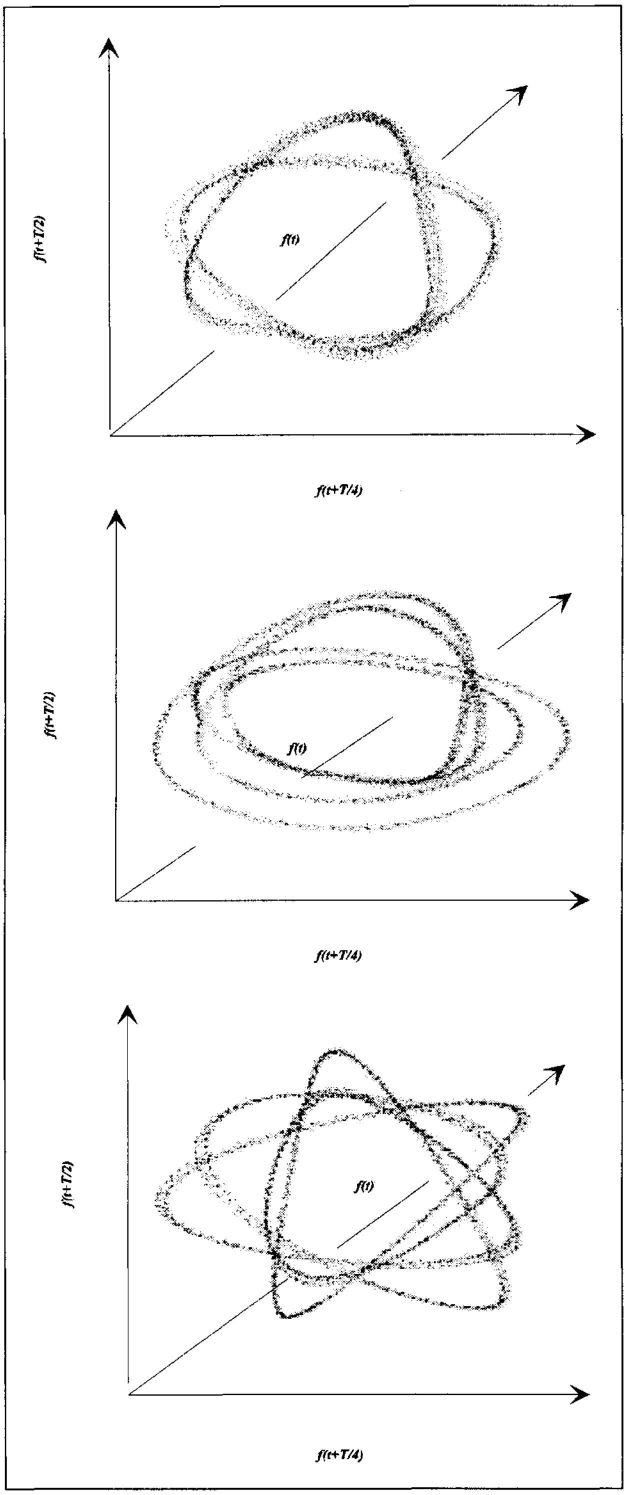

Figure 3. Phase Space Trajectories of parts of the signal analysed on Figure 1. The signals are chosen on the stable parts corresponding to the various bifurcations noted on Figure 1. a) can be interpreted as a periodic signal with a strong second harmonic, or as a first period doubling. It corresponds to the part of the Figure 1 marked '$2T$' b) and c) are obvious period doublings and correspond respectively on Figure 1 to the parts '4T' and '8T'.

rather soft plastic reed, and decreasing the blowing pressure after a normal sound, the crumhorn exhibits the same kind of period doublings as the trombone, for nearly all the fin-

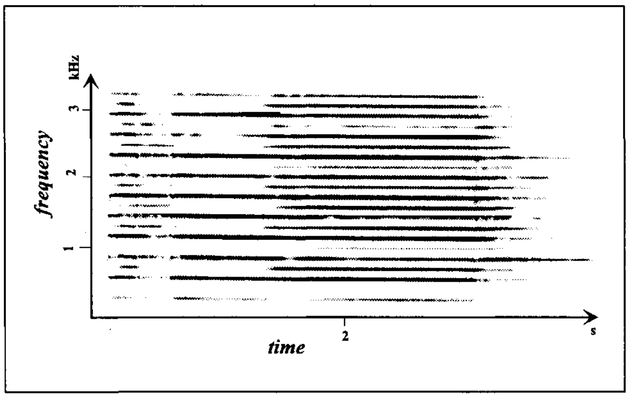

Figure 4. Sonagraphic representation of an E4 reed of Poitiers Cathedral Cliquot's Organ. The note is repeated and after a transient where components are visible at half the expected frequency, the pipe sounds at its frequency then bifurcates to the lower octave.

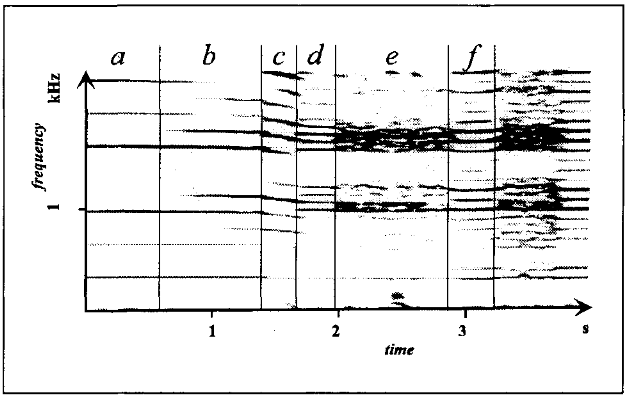

Figure 5. Sonagraphic representation of the period doubling cascade obtained on a crumhorn when the blowing pressure is continuously decreasing : a) normal periodic sound b) rise of subharmonic components indicating the first period doubling c) shift in frequency d) second period doubling e) possible chaotic zone f) period tripling

gerings (Figure 5). Moreover it is possible to identify a zone of unstable spectral characteristics after the second period doubling which may be chaotic. A clear period tripling follows this zone. As it is well known, the chaotic part of the bifurcation diagram is followed by a periodicity window of period three in a classical period-doubling scenario. It is then possible to conclude that, on this particular instrument, one obtains a quasi-complete period doubling cascade when decreasing the blowing pressure (which is the only possible control parameter). Two clear period- doublings, a "chaotic" or unstable zone, and a period tripling followed by another unstable zone are evident on the spectrogram. It is unfortunately very difficult, even with an artificial blowing system, to stabilise the 'chaotic' part. The cascade is obtained by continuously decreasing the blowing pressure. That means that the relative value of the pressure is closer and closer to the atmospheric pressure at each bifurcation. The pressure range where the signal is supposed to be chaotic is then very small and any variation of temperature or any movement around the instrument leads to the periodicity window. So we can-

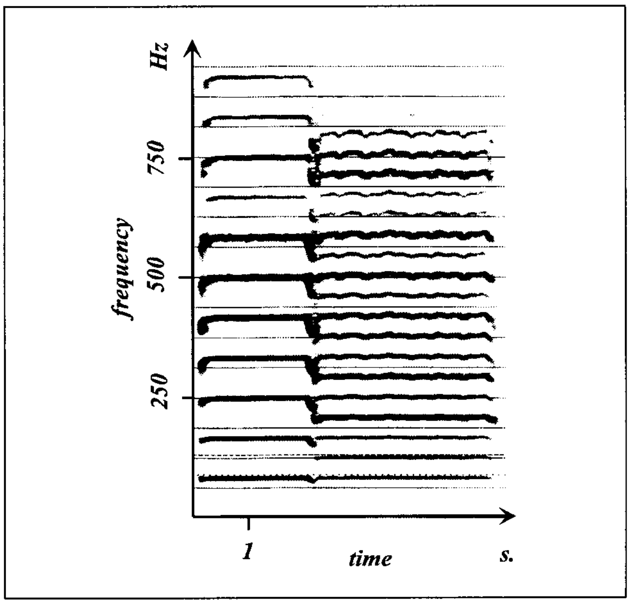

Figure 6. Sonagraphic representation of period doublings obtained on a bassoon whose reed has been specially designed to help this kind of production.

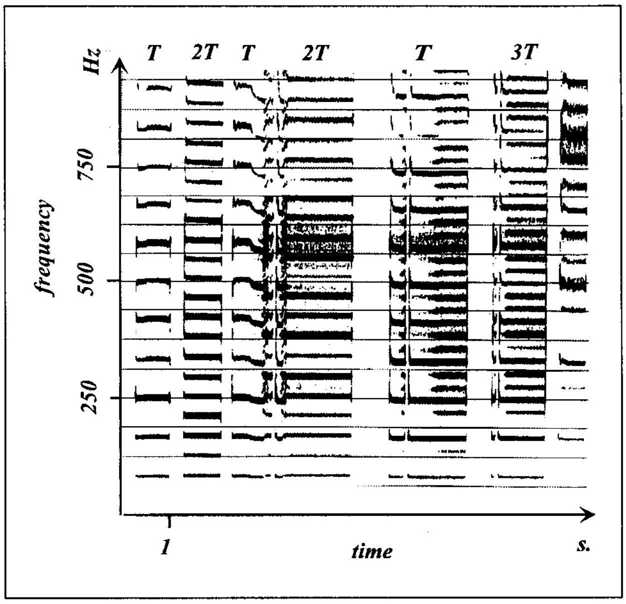

Figure 7. Same representation as on Figure 6 of period divisions obtained on the same bassoon but with a normal reed. Period doublings as well as period tripling are clearly visible.

not absolutely conclude the presence of chaos, even if this is the normal behaviour which takes place between a period doubling cascade and a period tripling. To our knowledge this cascade is the first complete one obtained in a musical instrument. One important feature is that it has been obtained through a blowing pressure decrease and using a soft reed, and that it can be produced by a musician or with a blowing machine.

Approximately at the same time we were working with the trombone and the crumhorn, we were in contact with a bassoonist who had designed a reed giving him the ability of playing in the contrabassoon range [35, 21]. His main idea had been to soften and enlarge the normal reed of the bassoon. The common way to obtain a softer reed is to make it carefully thinner with a cutter blade. The result is a very thin reed that looks like a contra-bassoon reed. The bassoon with this reed sounds very poor in the first regime, is hard to play in the second one but is able to produce sounds one and sometimes two octaves lower than those normally obtained with the fingering used. This phenomenon is possible on the whole first register of the instrument but calls for a skilled player. Again there is no resonance corresponding to the fundamental frequency. A careful examination of the signal in the time domain shows that the transition is of the same kind as for the trombone and the crumhorn. It corresponds to what is expected for a period doubling scenario: oscillation on a limit cycle of period T, then a first bifurcation leading to a period $2T$ and another bifurcation which may sometimes lead to a second period-doubling. This corresponds to a musical sound two octaves lower which is produced with increasing difficulty by the player! An analysis using a spectrogram (Figure 6) makes this period-doubling evident.

On bassoons, period doublings are not only possible with a specially designed reed. The same musician was able to produce them with a normal reed. The musical range where this phenomenon takes place is smaller than with the special reed. It is possible to obtain period doublings only on a few first register notes with an appropriate embouchure (Figure 7). From a technical and musical point of view the musician uses a relaxed embouchure where the reed is blown at low pressures. This is easily obtained on the 'large' reed and less easily on the 'normal' reed but the result is the same as for the crumhorn where under-blowing produces the controlled bifurcation cascade. The only difference is that, in the case of the crumhorn, there is an obvious and unique control parameter, the blowing pressure, which is not the case for the bassoon. The reed is mechanically softened making it thinner and larger, the blowing pressure is lowered but the other embouchure modifications are far from our ability to measure them.

4. Voice

Various models can describe the voice but the more recent approaches connect the physical behaviour of this very common physical system with the theory of Non Linear Dynamical Systems [36]. Their results in terms of stability suggest that it is possible to obtain various scenarios of chaotic transitions in such a system. Some experimental work and numerical simulations showing period-doublings in disordered voices are given in the references. Though they prove the possibility of the phenomenon, in the real cases presented (babies crying for example) there is no controlled and musical use of this possibility. We have carefully looked at known "abnormal" musical voiced occurrences and found some obvious period doublings. On a time domain representation as electroglottography (Figure 8) one can easily see the transition

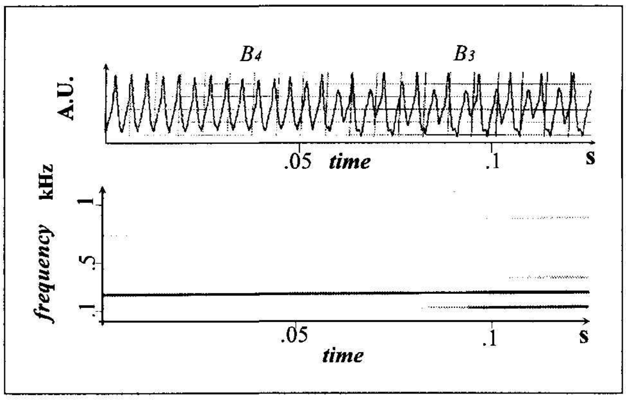

Figure 8. Time domain and sonagraphic representation of a period doubling obtained on a voice signal recorded by electroglottography. The bifurcation is clearly visible on the two representations.

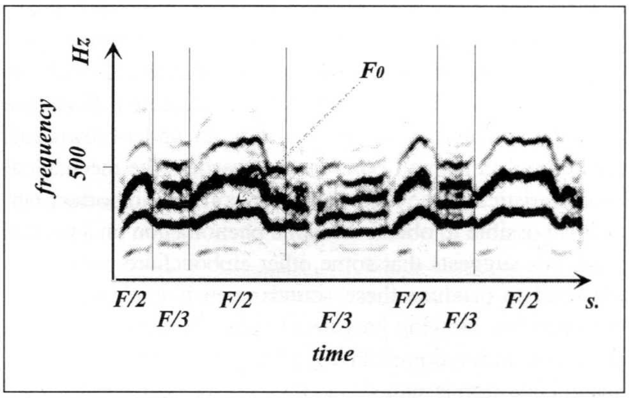

Figure 10. Various bifurcations in a recording of South African singers. The initial period is divided by two and even by three. The normal fundamental frequency is marked on the figure. Using period divisions the women can sing lower than 100 Hz.

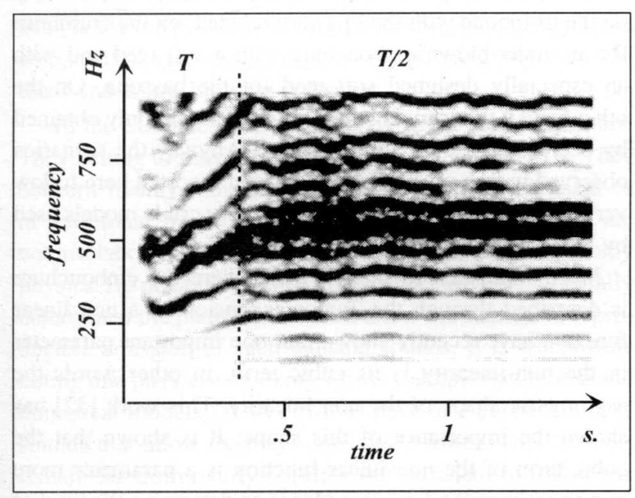

Figure 9. Period doubling on a record of traditional Tuva singers. There is not any energy in the lowest components of the voice during the part marked 'T' which correspond to a fundamental between 90 Hz and 125 Hz. The 'missing fundamental' obtained after the period doubling allows a frequency around 65 Hz.

on a demonstration done by a musician of a known Tibetan technique for singing with a low voice. In that particular example, one of the authors, who has learned and exercised this technique, sings a note (B3) and, through modification of her voice, jumps to the lower octave (B2). This shows that this special kind of 'passagio' is perfectly obtainable on singing voice, and not only on disordered voices. As in the bassoon case it has not yet been used as a musical effect in classical western music but is known, understood and can be used by a singer as a possible technical gesture which is of normal use in Tibetan music. This has been the case for 'normal multiphonics' for years. They were first employed in traditional music or jazz and only included later in the normal classical contemporary musical language.

Concerning multiphonic production, traditional music presents a great number of interesting features from an acoustical point of view. Another example is given here. A recording has been extracted from a traditional singing performance of Asian Tuva singers (Figure 9) [37]. The singers exhibit very low pitch. On the sonagraphic representation one can easily see that after a first melodic glissando of fundamental frequency going from 90 Hz to 125 Hz a bifurcation appears with an obvious division by two of the fundamental frequency giving a new fundamental around 65 Hz. As in the former example, the signal corresponds to a bifurcation leading to a sound one octave lower. This stable period-doubling seems to be very often used in this way to obtain unusual low pitches, particularly in this region not so far from Tibet where it is known as Kangiraa style (where Kangiraa means: 'to speak in a husky voice').

Another traditional music exhibits the same kind of phenomena. The South African women of the Xhosa ethnic group [38] used a special and very peculiar singing style known as "ordinary Umngqokolo". On the recorded sounds one finds first the melodical fragment sung with a normal voice (mean pitch around 200 Hz) by a woman, then the "Umngqokolo" version where the perceived pitch is divided by two or three. The spectrogram (Figure 10) presented here makes evident this well-controlled technique showing subtle alternations between period doubling and period tripling. Another style, "umngqokolo ngomqangi" produces a constant period doubling on two different fundamental frequencies, a tone apart, as for musical bow playing. Such singing technique with period doubling (or tripling) produces fundamental frequencies in a rather low range for women, between 70 Hz and 130 Hz, and enable them to produce spectral melody on higher harmonics, through mouth resonances.

5. Period doubling production and the Non Linear part of wind instruments

Such evidences of period doubling scenarios performed by musicians and singers, and included in musical sequences, raise many questions. It is beyond the scope of this paper to give theoretical answers to these questions. We only want to give some hints that could help to better understand how these phenomena occur.

First, how does the musician produce these sequences? There are partial answers coming from the trombone productions where the musician has a very relaxed embouchure. Some other useful features can be extracted from the crumhorn behaviour where one uses an under-blown soft reed closed in a small box. On the bassoon, the mechanical characteristics of the reed (large and soft) are important but, as it is possible to obtain the same phenomenon on a normal reed, this suggests that some other embouchure parameters are used to produce these sounds even if one finds again the same low blowing pressures to play these sounds. Again the musician is recommending a very relaxed embouchure. A second question is: can we obtain such signals on single-reed instruments such as clarinets or saxophones? Up to now, all reported results concern double-reed systems (lips and vocal folds are double-reeds). To try to answer we have built a very thin reed for a clarinet. This reed is as thin as a sheet of paper and has been obtained from a soft reed (Vandoren no.2) keeping the same profile but making it as thin as possible. The instrument is no longer playable in a "normal" way, but with an extremely soft blowing and a loose embouchure it is possible to play it on the first register with a terribly bad timbre. Then by decreasing carefully the blowing pressure an unstable period doubling can obtained. Skilled players are able to sustain it. It has not been possible to do the same on a saxophone. All these preliminary proofs show that period doublings can be obtained provided three conditions are fulfilled at least: 1) an extremely soft reed or relaxed vocal folds or lips, 2) low blowing pressure for the normal sound, 3) a decrease of the blowing pressure to obtain the period doubling.

The sound production in woodwinds involves at least three important parameters: the woodwind resonances defined by the bore impedance, the nonlinear part which corresponds to the effect of the reed on the upstream flow and the control parameter defined by the musician's embouchure. The later may include vocal tract resonances as well as mechanical parameters such as damping, reed resonance frequency etc. An experienced musician can play on all the fingerings of a clarinet or a saxophone not only periodic sounds but also multiphonics of various timbres changing his embouchure from loose to tight. One can vary, only through embouchure control, from periodic sounds to a wide variety of multiphonics. This is well known, for example, on the most atypical note of the saxophone, the medium G fingering, where small changes in the lower lip position may lead to various sounds between the two first registers. It seems to indicate that the understanding of the whole behaviour of woodwinds lies not only in the resonance curves but also in the knowledge of the embouchure parameters. One may think that the musician knows how to choose the shape of the non-linear function to produce one sound or another. 'Octave multiphonics' or period doublings do not differ from other behaviours of a woodwind. One may think they are the consequence of an interaction with the vocal tract producing satellite frequencies exactly at half the frequency of each peak of the periodic normal sound. It may have been possible on the trombone and on the bassoon where the musicians can adjust the resonances of their vocal tract, but not on the crumhorn where one cannot adjust anything and where it is possible to produce period doubling on nearly all the fingerings.

Is it possible to introduce these features in a general theoretical scheme of sound production in woodwinds including this special kind of multiphonic?

Some partial answers are available in the literature and particularly in [39, 30, 29, 12, 40, 41].

The most well known period doubling scenario involves the iteration of a quadratic function like $y = \lambda x(1-x)$ [16]. An acoustical equivalent of this scenario has been obtained on electro-acoustic systems mimicking musical instruments by increasing the gain λ which is taken as the control parameter of the system [18] for the quadratic function. Increasing the gain (λ) corresponds to sharpening the shape of the non-linear function used to represent the effect of the reed. This strictly corresponds to a "loose embouchure" as defined by Backus [30]. So it is not surprising to obtain such a cascade on the trombone with the lips very relaxed, on the crumhorn for an under-blown embouchure with a soft reed and with an especially designed soft reed for the bassoon. On the other hand it is surprising that the cascades are only obtained by decreasing the blowing pressure. Anyway the scenarios observed in true musical situations and reported here follow very precisely those predicted by the very crude models used by Maganza *et al.* [2].

More sophisticated models [42] where the embouchure is described through the Taylor expansion of a non linear function have recently shown that one important parameter in the non-linearity is its cubic term, in other words the asymmetric shape of the non linearity. This work [32] has shown the importance of this shape. It is shown that the cubic term of the non linear function is a parameter more important than the bore impedance to determine the kind of bifurcation (direct or inverse) that is obtained by changing the control parameter. For non-linear functions that are too symmetrical, the result is not typical of a real instrument. The basic description of a woodwind involves two equations: a linear one with delays and a non-linear instantaneous one (see MacIntyre 83 for more details). The behaviour of this model is extremely rich. One can derive from it numerical models including non-linear differential delayed equations and phenomenological models [7, 40, 41]. These models, widely used in fluid mechanics, show that it is perfectly possible to obtain chaos following a period doubling cascade as well as quasi-periodic scenarios by changing the shape of the non-linear function. They have the advantage to use only one variable, the acoustic pressure at the bore entrance section. Taking the blowing pressure as control parameter and decreasing it, they allow the simulation of period doubling scenarios. Unfortunately these models are phenomenological ones and it is difficult to connect all the numerical parameters with physical parameter. However they are able to reproduce results close to what we measured on the crumhorn.

6. Conclusion

The period doubling scenario, often known as Feigenbaum scenario, has been described in this paper for woodwinds. The identification of such a scenario in natural musical sounds allows a more complete description of their general behaviour. For obtaining such peculiar sounds on woodwinds the embouchure seems to be one of the main control parameters. This control parameter acts on the shape of the non-linear function linking the acoustic pressure to the flow at the input section of the instrument. To obtain period doublings on double reeds and lip reeds it seems necessary to soften the reed or/and to relax the embouchure which must be blown very softly, that means with a blowing pressure lower than usual. The fact that period doublings have never been observed before on single reed instruments with normal reeds and that they can be observed with a clarinet reed reduced to a thin blade of cane confirms this point of view. This allows us to relate the period doubling effect mainly to a very loose embouchure, something which is only possible with very soft reeds and at low blowing pressures.

All the examples presented here share a common aspect. They belong to musical performance or are close to it. They concern mainly musicians, highly educated in classical or in traditional music. They show that period doublings are not just laboratory experimental phenomena concerning only physics. By practice and will, musicians have experimentally determined the parameters which allow the stable and reproducible emission of such unusual sounds. It is worthwhile noting that physicists looking for very simple theoretical and numerical models for woodwinds have first described these sounds that allow to 'play' one octave lower. Their models remain far from reality but they are sufficient to follow the scheme we have obtained experimentally, for example, on the crumhorn. In some sense many models, excepting that of Mac Intyre *et al.* [1] but including that of Maganza *et al.* [18], can be understood as phenomenological ones. This means that they are built on very simple equations where the parameters cannot really be related to the physical parameters of the studied system but where the behaviours obtained follow very precisely those reported in experiments. It is very interesting to note that other divisions, mainly by 3 or 5 of the fundamental frequency that can be theoretically obtained during a period doubling scenario and are not often found in experimental literature, are sometimes obtained by musicians. We have reported here divisions by 3. Some divisions of the fundamental frequency by 3 or by 5 have been experimentally obtained on bowed strings. This may be related to the same scenario as well as to wolf note production [13]. Nevertheless it seems difficult for a musician to control the whole 'glissando' from period 1 to period 3 through the period-doublings and the chaos preceeding the periodicity window. In the absence of period doublings followed by period tripling it is not possible to conclude that the divisions by 3 or by 5 belong to a period doubling cascade. They may also be the result of a quasiperiodicity scenario with phase locking and a ratio 1:3 or 1:5 as has been demonstrated by Puaud *et al.* [13]. Anyway, we have shown that more than successive period-doublings are possible on various classical instruments. This means that this kind of 'octave multiphonic sound' belongs to the normal and musical range of, at least, woodwinds.

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